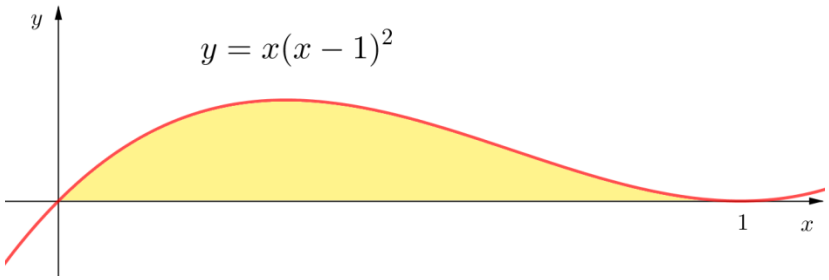
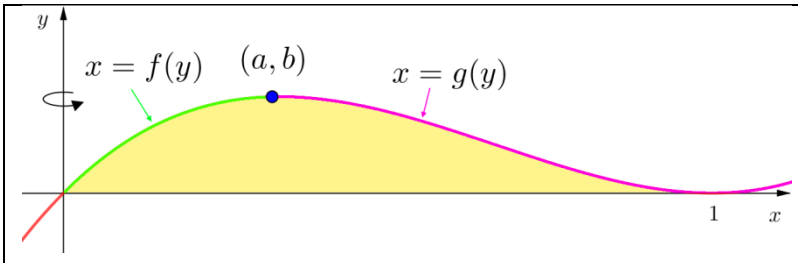
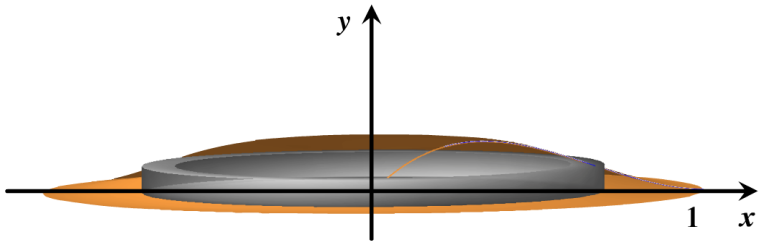
1) Let S be the solid obtained by rotating the region shown in the figure about the y-axis. Explain why it is awkward to use slicing to find the volume V of S. Sketch a typical approximating shell. What are its circumference and height? Use shells to find V.





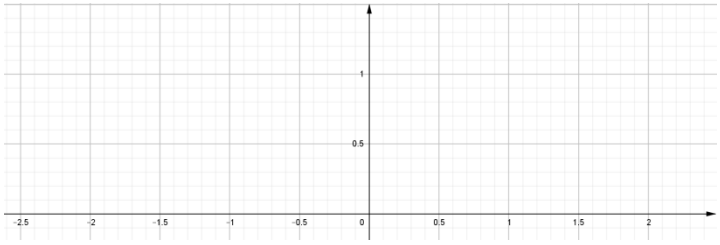
By using the "washer" method, we would first have to locate the local maximum point (a,b) of $y = x(x-1)^2$. Then we would have to solve the equation $y = x(x-1)^2$ for x in terms of y to obtain the functions $x = f(y)$ and $x = g(y)$. This would be difficult because it involves the cubic formula. We would then find the volume by using $V = \pi\int_0^b \left\{\left[g(y)\right]^2 - \left[f(y)\right]^2\right\} dy$.



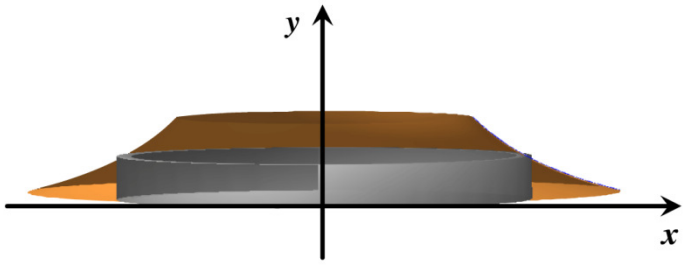
Using shells, a typical approximating shell has radius x. So its circumference is $2\pi x$. Its height is $y = x(x-1)^2$. So its volume is $\frac{\pi}{15}$.

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis. Sketch the region and a typical shell.

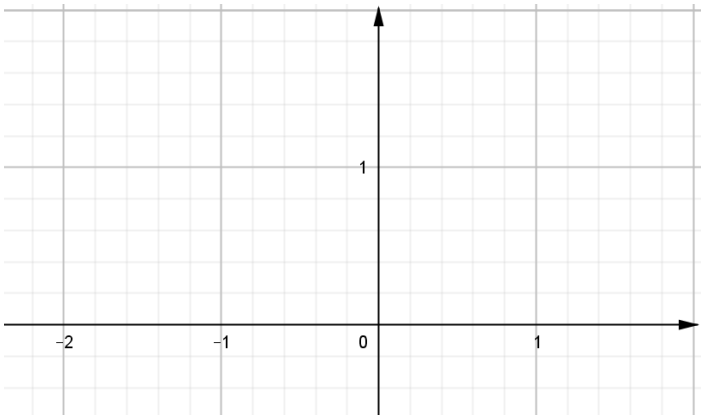
2) $y=\frac{1}{x}, \quad y=0, \quad x=1, \quad x=2$



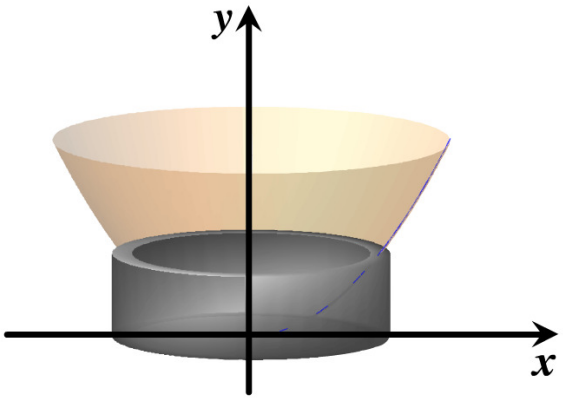
2π



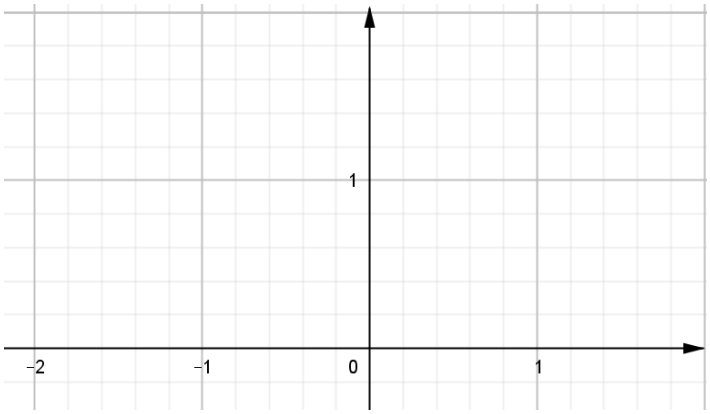
3) $y=x^2, \quad y=0, \quad x=1$



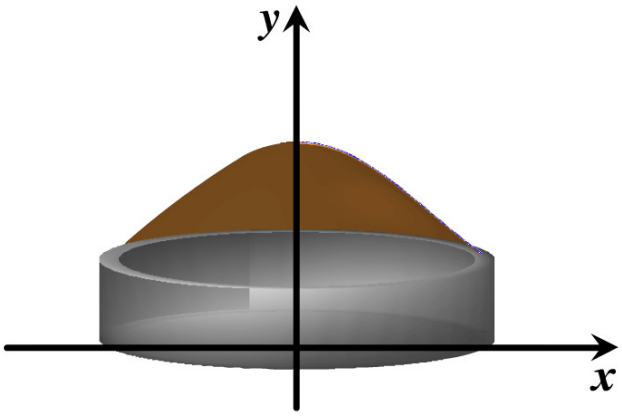
$\frac{\pi}{2}$



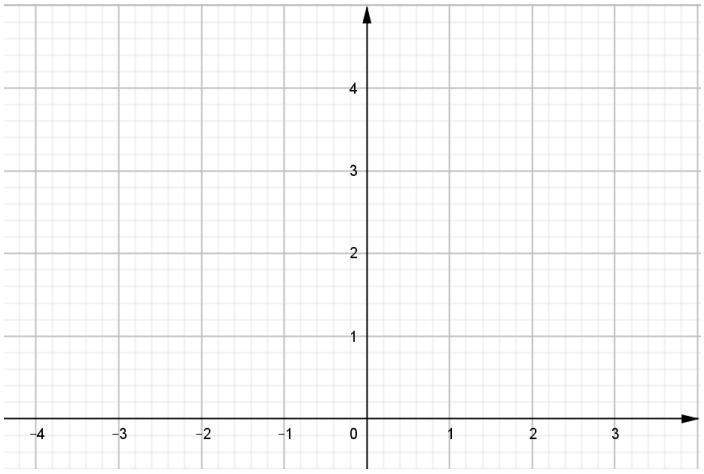
4) $y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 1$



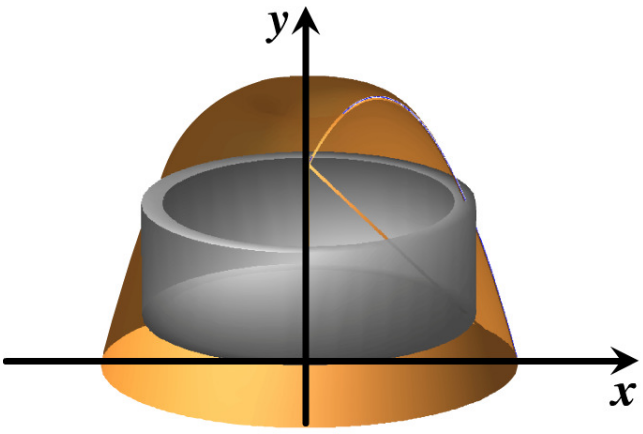
$\pi\left(1-\frac{1}{e}\right)$



5) $y = 3 + 2x - x^2, \quad x + y = 3$

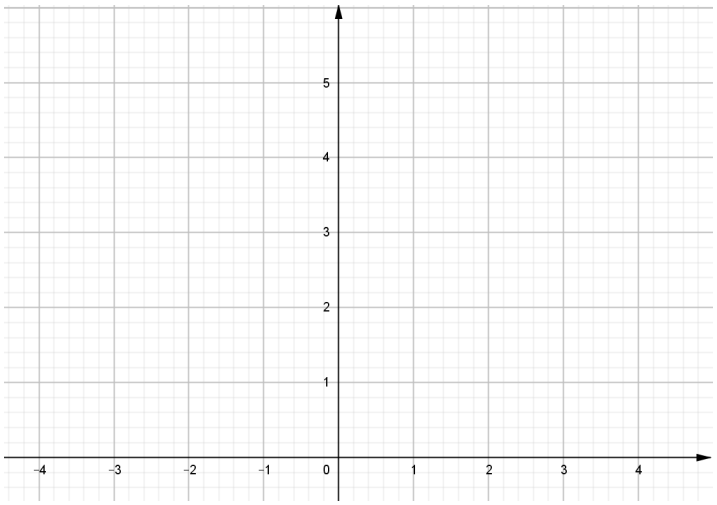


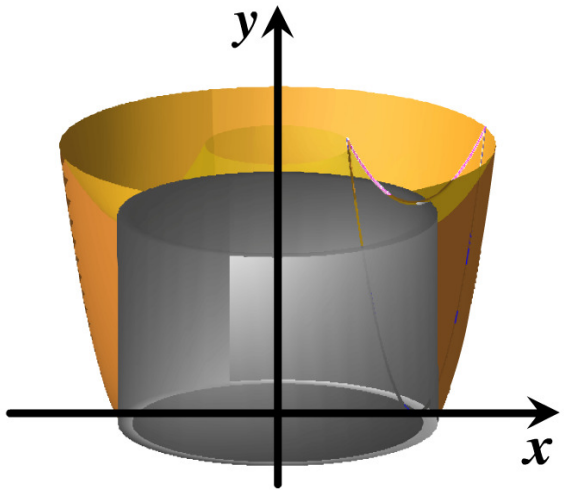
$\frac{27\pi}{2}$



6) $y = 4(x-2)^2, \quad y = x^2 - 4x + 7$

$\boxed{16\pi}$

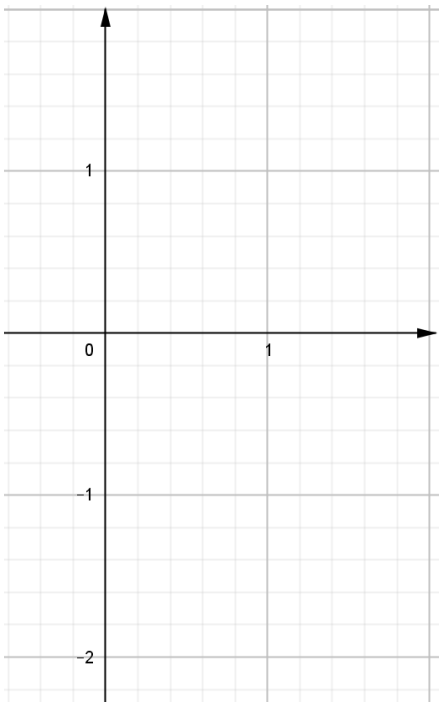


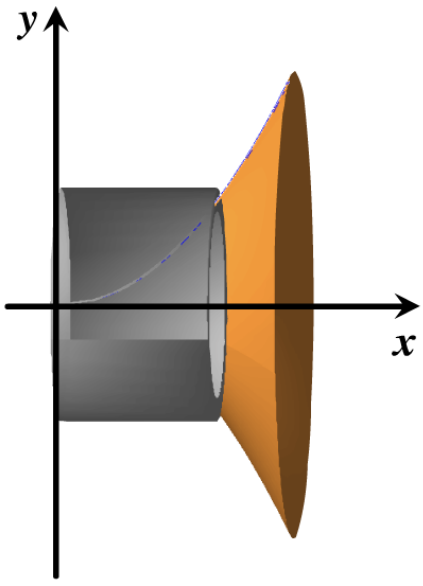


Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Sketch the region and a typical shell.

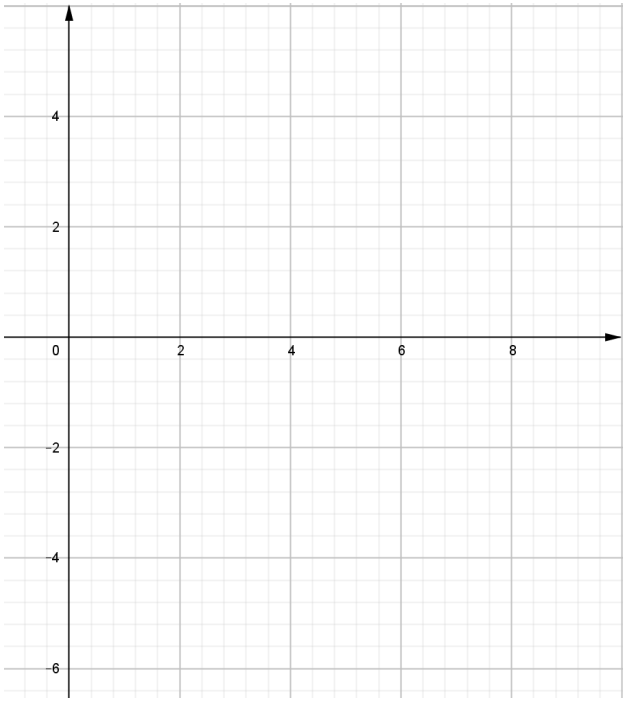
7) $x = \sqrt{y}, \quad x = 0, \quad y = 1$

$\boxed{\frac{4\pi}{5}}$

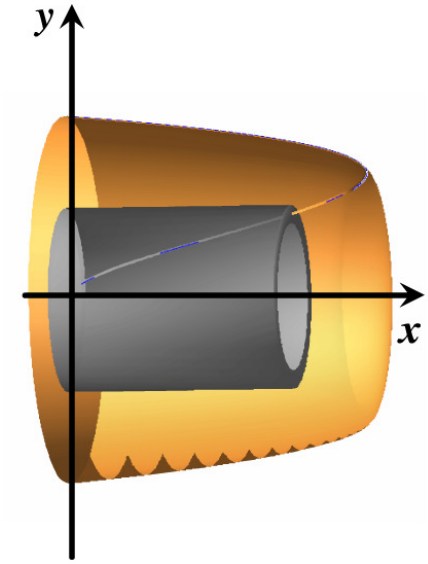




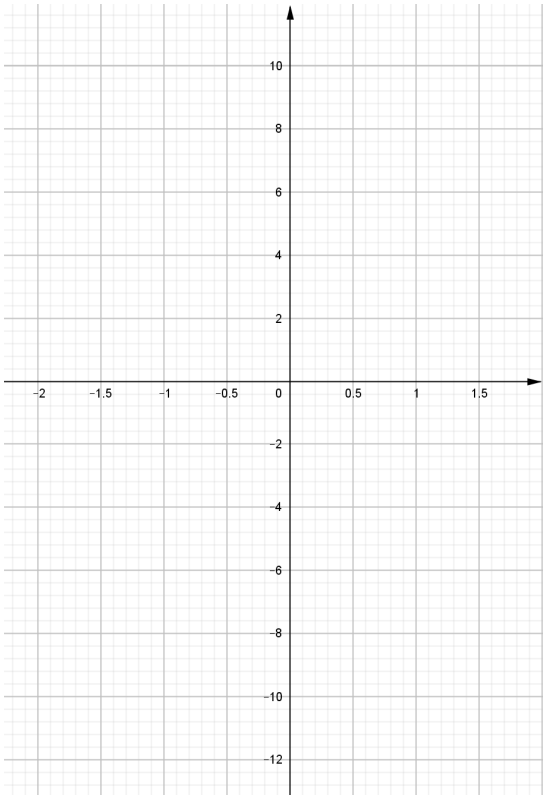
8) $x = 4y^2 - y^3, \quad x = 0$



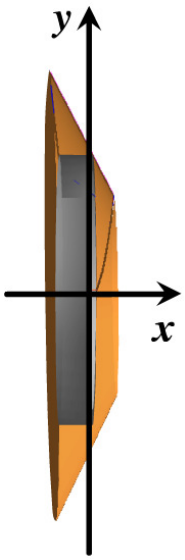
$\boxed{\frac{512\pi}{5}}$



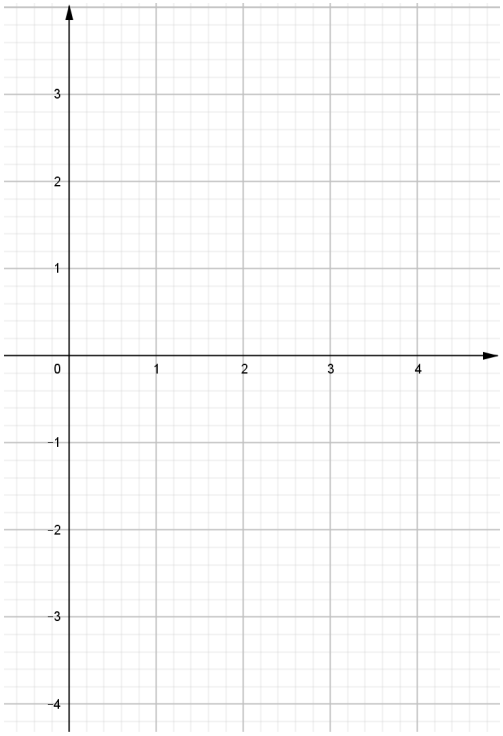
9) $y = 4x^2, \quad 2x + y = 6$



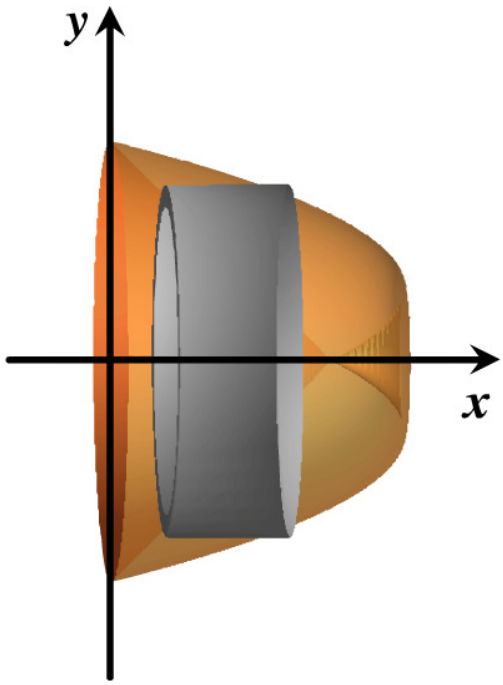
$\boxed{\frac{250\pi}{3}}$



10) $x+y=3, \quad x=4-(y-1)^2$

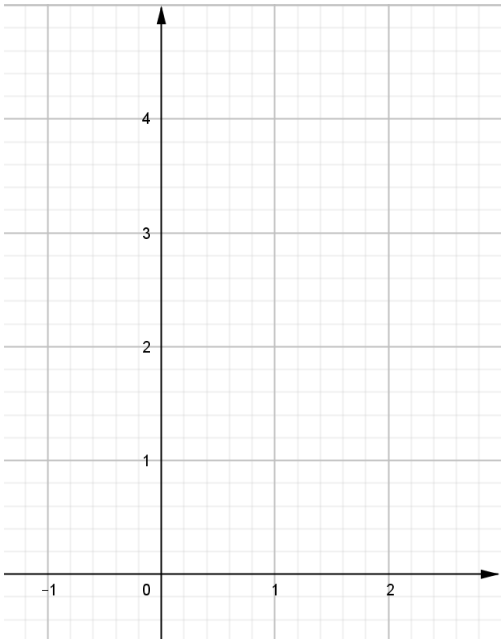


$\boxed{\frac{27\pi}{2}}$

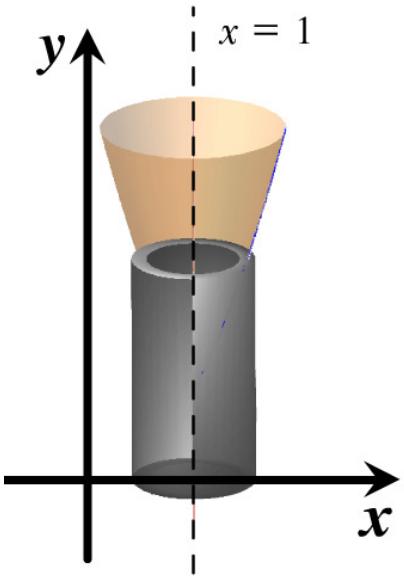


Use the method of cylindrical shells to find the volume generated by rotating the region bonded by the given curves about the specified axis. Sketch the region and a typical shell.

11) $y=x^2, \quad y=0, \quad x=1, \quad x=2$ | about $x=1$

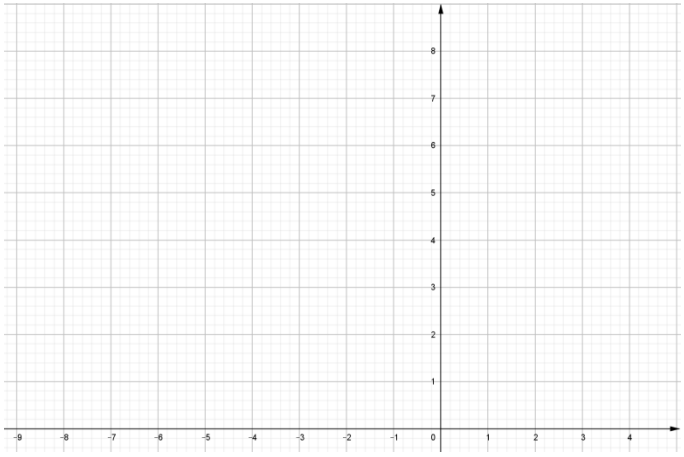


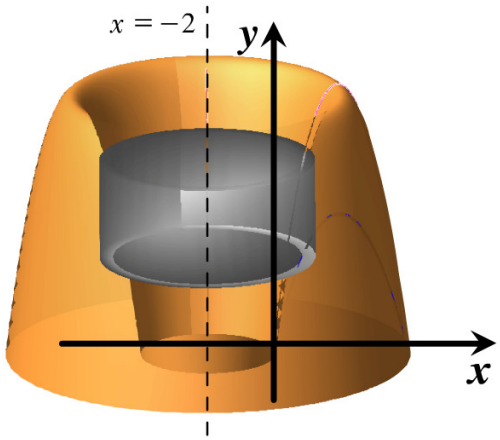
$\boxed{\frac{17\pi}{6}}$



12) $y = 4x - x^2, \quad y = 8x - 2x^2$ | about $x = -2$

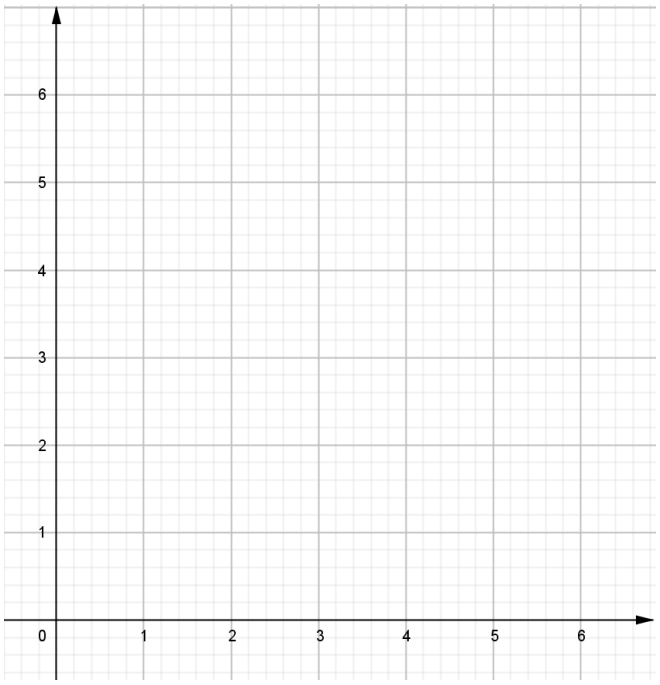
$\boxed{\frac{256\pi}{3}}$

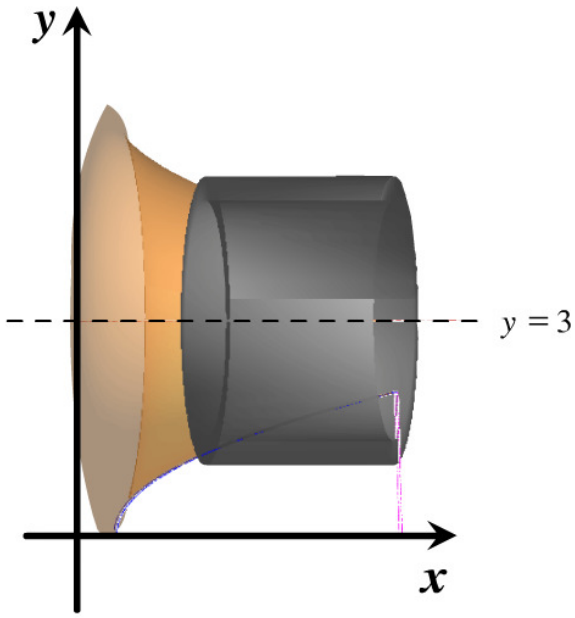




13) $y = \sqrt{x-1}, \quad y = 0, \quad x = 5$ | about $y = 3$

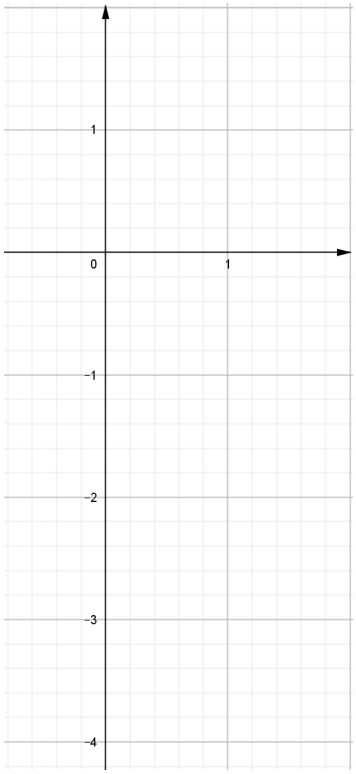
$\boxed{24\pi}$

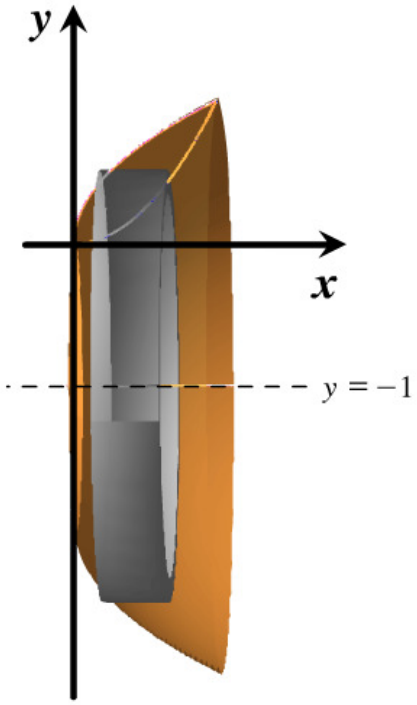




14) $y = x^2, \quad x = y^2$ | about $y = -1$

$\boxed{\frac{29\pi}{30}}$





Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

15) $y = x, \quad y = 4x - x^2$ | about $x = 7$ $\boxed{V = \int_0^3 2\pi(7-x)\left[(4x-x^2)-x\right]dx}$

16) $x^2 - y^2 = 7, \quad x = 4$ | about $y = 5$ $\boxed{\int_{-3}^{3} 2\pi(5-y)\left(4-\sqrt{y^2+7}\right)dy}$

17) Use the Midpoint Rule with $n = 4$ to estimate the volume obtained by rotating about the y-axis the region under the curve $y = \tan x$, $0 \le x \le \dfrac{\pi}{4}$.

$$\boxed{V \approx 1.142}$$

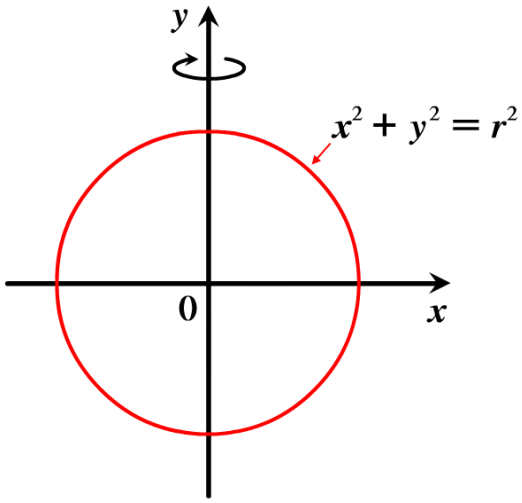
18) Use a graph to estimate the x-coordinates of the points of intersection of the given curves. Then use this information to estimate the volume of the solid obtained by rotating about the y-axis the region enclosed by these curves.

$$y = x^4, \quad y = 3x - x^3$$

$$\boxed{V \approx 4.62}$$

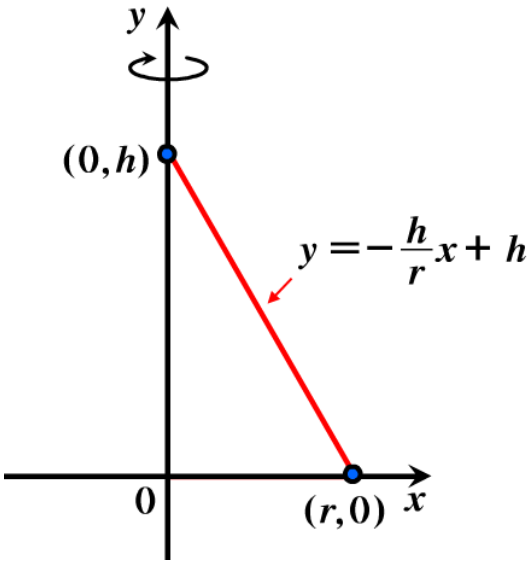
Use cylindrical shells to find the volume of the solid.

19) A sphere of radius r. Use the following diagram to find the volume by using calculus.



$$V = \frac{4}{3}\pi r^3$$

20) A right circular cone with height h and base radius r. Use the following diagram to find the volume by using calculus.



$$V = \frac{\pi r^2 h}{3}$$